Operational management of food delivery markets: from tactical to network equilibrium

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Part I – a stylized model

Tactical operations of service region dimensioning, bundling, and matching for on-demand food delivery services

With Jintao Ke, Hai Wang, and Yafeng Yin. 2025. *Transportation Research Part C: Emerging Technologies 174*, 105069.

Background and importance

On-demand food delivery tends to be the largest among urban on-demand mobility services.

- OFD users in China reached 43.52%, surpassing ride-sourcing, the largest in the shared economy[1]
- Number of OFD workers in the US has increased by 3 million from 2018 to 2021[2]

[1] State Information Center. (2021, February). China sharing economy development report.

[2] Garin, A., Jackson, E., Koustas, D., & Miller, A. (2023). The evolution of platform gig work, 2012-2021. National Bureau of Economic Research, (w31273)

Food delivery vs ride sourcing

It is different from ride-sourcing services:

- More time slack, because food is less needy than human passengers
- Order consolidation, because food needs less space
- Localized service, compared with ride-sourcing

Related works and contribution

- Yildiz and Savelsbergh (2019)[3]: service area and equilibrium
- Bahrami et al. (2023). (2023)[4]: bundling and equilibrium
- Ye et al. (2024b). (2024)[5]: bundling and equilibrium (different physical model)
- Ke et al. (2024)[6]: bundling, equilibrium, and restaurant preparation delay
- **This work**: Service area, bundling, equilibrium, and restaurant preparation delay

[3] Yildiz, Baris, and Martin Savelsbergh. (2019). *TR-C*, 100, 177–199.

[4] Bahrami, Sina, Mehdi Nourinejad, Mahmood Mahmoodi Nesheli, and Yafeng Yin. *TR-E*, 179, 103313.

[5] Ye, Anke, Kenan Zhang, Xiqun (Michael) Chen, Michael G. H. Bell, Der-Horng Lee, and Simon Hu. *TR-E*, 187, 103597.

[6] Ke, Jintao, Ce Wang, Xinwei Li, Qiong Tian, and Hai-Jun Huang. (2024). *TR-E*, 184, 103467.

Motivation

1. How does the transportation side of this service impact the entire system? Driver routing.
2. What are the key factors that impact driver routing? Service dimension and bundle size.
3. How to analyze the routing part? Batch matching $\rightarrow$ VRPs $\rightarrow$ continuum approximation (differentiable, analytical tool) $\rightarrow$ matching equilibrium.

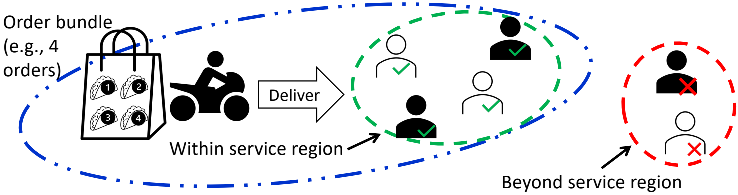
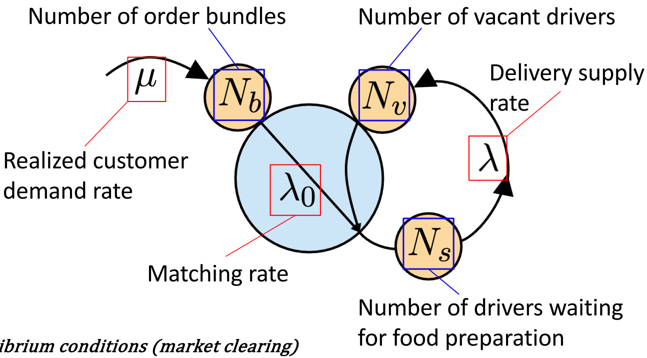


Figure: An illustration of bundling and service region

Equilibrium analysis

Equilibrium is established as the conservation of demand/supply flows using Little's law, from which we solve for N_v and N_b



Equilibrium conditions (market clearing)
all flows are equal

Formulations

Customer total waiting time (W_c) is decomposed into five analytical components

$$W_c = \underbrace{W_{c,a}}_{\text{Accumulation}} + \underbrace{W_{c,q}}_{\text{Matching}} + \underbrace{W_p + W_{c,d}}_{\text{Pickup and delivery}} + \underbrace{W_{fp}}_{\substack{\text{Food}\\\text{preparation}}} \quad (1)$$

1. Accumulation time $W_{c,a}$: # orders / demand
2. Matching time $W_{c,q}$: based on matching probability
3. Delivery $W_{c,d}$ and pickup time W_p: routing approximation
4. Delay due to food preparation W_{fp}: time difference

These values vary across different demand (N_b) and supply statuses (N_v) $\rightarrow$ Solve for them

Accumulation time

- The waiting time before a bundle is formed
- Customer demand

Total number of merchants — Maximum delivery distance

$$q = \sum_{i=1}^{M} \int_0^R \delta_i(r) 2\,\pi r\, \mathrm{d}r$$

Realized customer demand — Demand density rate

- Customer's accumulation time

$$W_{c,a} = \frac{(k-1)/2}{q/\bar{N}_w}$$

Realized customer demand for each bundle (driver)

Considering the worst (wait for $k-1$ orders) and the luckiest cases (no waiting)

Matching time

- Matching probabilities p_c and p_v for customers and drivers, respectively.
- Conservation of the number of matched order bundles/vacant drivers:
  - $p_c N_b = p_v N_v$,
  - N_b: number of bundles, N_v: number of vacant drivers.
- Matching time is from competition
- Customers' and drivers' expected matching times are

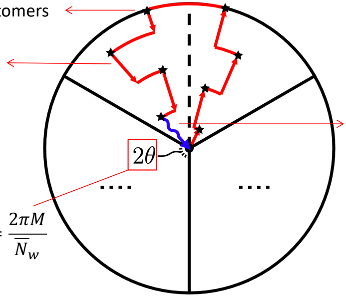
$$W_{c,q} = \frac{1}{2}\tau p_c + \frac{3}{2}\tau p_c(1-p_c) + \cdots = \tau\left(\frac{1}{p_c} - \frac{1}{2}\right), \quad (2)$$

$$W_{v,q} = \frac{1}{2}\tau p_v + \frac{3}{2}\tau p_v(1-p_v) + \cdots = \tau\left(\frac{1}{p_v} - \frac{1}{2}\right). \quad (3)$$

Delivery time

We approximate the delivery-and-return routing using continuum approximation (Newell and Daganzo, 1986)

$$W_{v,d} \approx \frac{L_{\text{line-haul}} + L_{\text{transversal}}}{v}$$

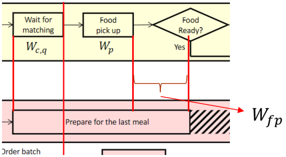


$$2\theta = \frac{2\pi}{\overline{N}_w / M} = \frac{2\pi M}{\overline{N}_w}$$

Avg number of working drivers

Figure. Illustration of the delivery routing approximation

Waiting time due to food preparation



- Before the last order is being cooked, there is customers' matching time $W_{c,q}$ and return-pickup time W_p

$$W_{fp} = \max\left(\tau_M - \left(W_{c,q} + W_p\right), 0\right)$$

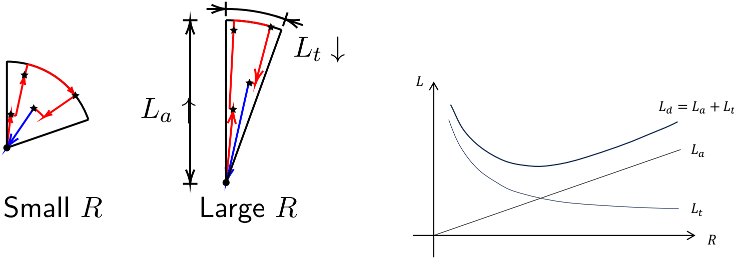
Average meal preparation time of one meal

Matching plus return-pickup time

Properties of the OFD system—R and $W_{c,d}$

Proposition 2.2 in paper

Customers' average delivery time $W_{c,d}$ first decreases and then increases with R.



(a) Illustration of the impact of an increase in R

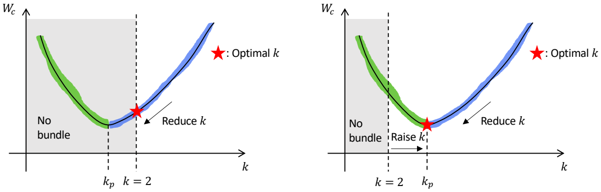
(b) Different trends of the axial and transversal travel distances

Figure: Depiction of insights into the maximum delivery distance.

Properties of the OFD system—k and W_c

Proposition 2.4 in paper

In the RP-SD scenario[7], customers' total waiting time monotonically increases with k when R is small, and first decreases and then increases with k when R is large.



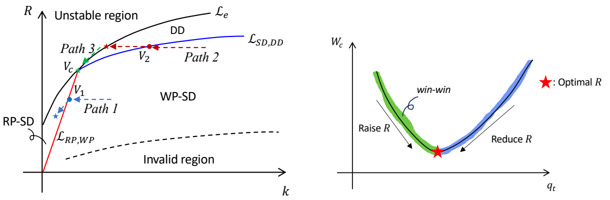
(a) Bad to increase k (b) Good to raise k (small k)

[7] $W_{fp} = 0$, supply dominant

Properties of the OFD system—Operations

Proposition 3 in paper

Under market equilibrium, W_c^* always happens in the RP-SD scenario[8] (including its boundaries); q_t^* requires largest possible R.



(a) Paths of decreasing W_c (b) The win-win scenario

[8] $W_{fp} = 0$, supply dominant

Simulation for driver routing

The simulation exhibits uncertainty and to see the trend of $W_{c,d}$ with respect to R, we offset those $W_{c,d}$ under large R using the $W_{c,d}$ under the smallest R, and draw the curve of $W_{c,d}$'s v.s. R's. For example, if the smallest R in the trial set is $R = 2$ km and we have its associated delivery time $W_{cd,R=2}$, then for $R = 3$ km we also have $W_{cd,R=3}$, and so do later R's. Then on the graph, we show $\{0, W_{cd,R=3} - W_{cd,R=2}, W_{cd,R=4} - W_{cd,R=2}, \cdots\}$ v.s. $\{2, 3, 4, \cdots\}$.

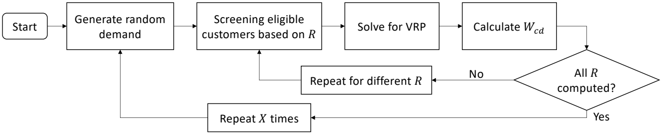
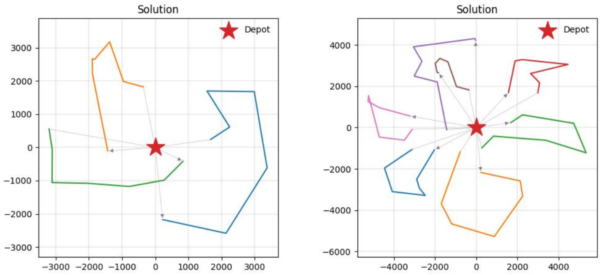


Figure: Simulation logic

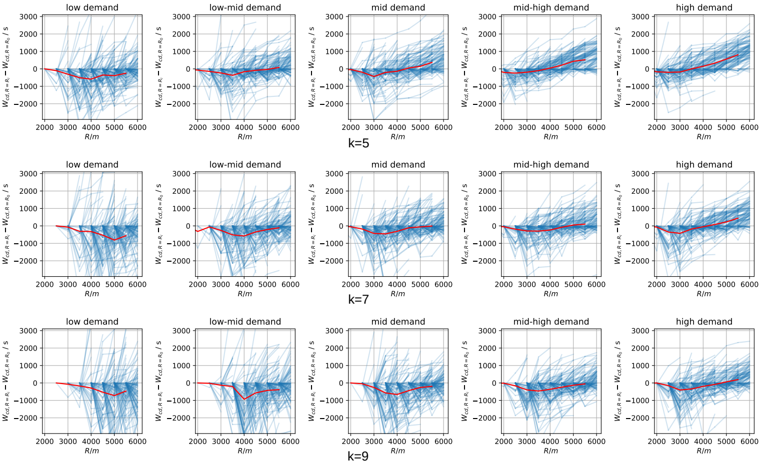
Simulation for driver routing

Example



LHS: $R = 3500$ m, RHS: $R = 5500$ m. We can see that the sectors on LHS are fat while on RHS they are thin.

Simulation for driver routing



A three-sided network equilibrium model for on-demand food delivery services

With Jintao Ke and Xiaolei Wang. *Transportation Research Part B: Methodological 209*. 103461.

Related works and contribution

Related works:

- In recent CAV network models, people[9] overlooked the idle cruising
- In ride-sourcing[10], there is no demand consolidation
- The most classical service network model[11] simplifies the matching using aggregate metrics

Contribution:

- Status transition of drivers as a loop
- Demand consolidation (order bundling)
- Matching and dispatching problems explicitly as mathematical programs and develop a solution approach

[9] Hou, Jiaxin, Kexin Wang, Ruolin Li, Jong-shi Pang. 2025. Preprint at Arxiv.

[10] Xu, Zhengtian, Zhibin Chen, Yafeng Yin, Jieping Ye. 2021. TS, 55 (6).

[11] Hai Yang, Cowina W.Y. Leung, S.C. Wong, Michael G.H. Bell. 2010. TR-B, 44 (8–9).

A leader-follower game

A bi-level (Stackelberg) game structure

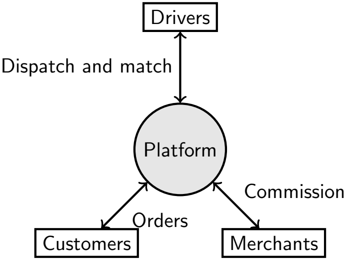
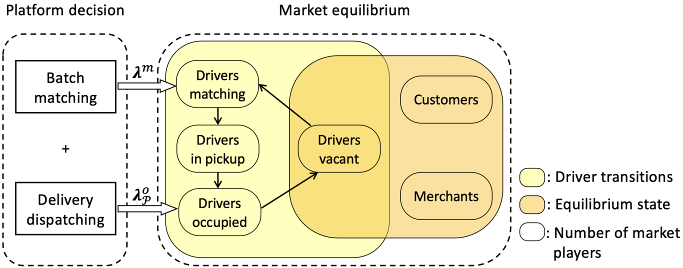


Figure: The three-sided market

In a network where its vertices represent catchment areas,

- How does demand distribute across the area
- How do vacant drivers cruise through the network
- How do market participants behave

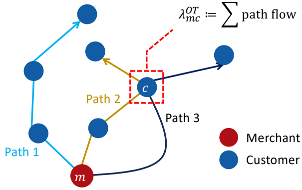
A leader-follower game



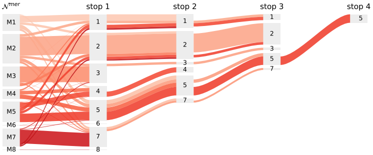
Market players' behaviors modeled by random utility theory

- Customer demand: merchant density, distance, delivery fare, matching time, delivery time.
- Vacant driver cruising: merchant density (attractiveness), earnings, distance, matching time, service time.
- Driver entry: hourly earnings.
- Merchant join: driver availability, commission rate.

How we deal with bundling delivery



(a) Flux as the total supply flow



(b) Sample path flow distribution

- Pre-calculate all path combinations; starting from merchant, to customer nodes in all possible combinations
- Assign flows (continuous) as decision variables to each path

The stationary matching and dispatching

$$\min_{\mathbf{m},\boldsymbol{\lambda}^m,\boldsymbol{\lambda}^o_{\mathcal{P}}} \overbrace{f_m(\mathbf{m},\boldsymbol{\lambda}^m)}^{\substack{\text{Matching:}\\ -\text{revenue}+\text{reposition cost}}} + \overbrace{f_d(\mathbf{m},\boldsymbol{\lambda}^o_{\mathcal{P}})}^{\substack{\text{Delivery:}\\ \text{dispatching cost}\begin{cases}\text{1. no intervene}\\ \text{2. min cost}\\ \text{3. min waiting}\end{cases}}} \tag{4a}$$

$$\text{s.t.}\quad \text{Equilibrium constraint:}\quad \mathbf{m} \in \mathcal{S}_e\left(\boldsymbol{\lambda}^m, \boldsymbol{\lambda}^o_{\mathcal{P}}\right), \tag{4b}$$

$$\text{Matching feasibility:}\quad \boldsymbol{\lambda}^m \in \mathcal{F}_m(\mathbf{m}), \tag{4c}$$

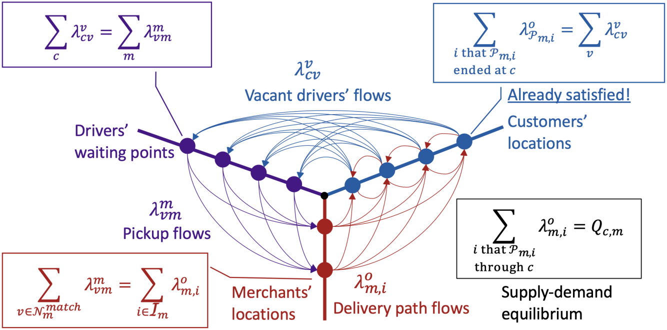
$$\text{Dispatching feasibility:}\quad \boldsymbol{\lambda}^o_{\mathcal{P}} \in \mathcal{F}_d(\mathbf{m},\boldsymbol{\lambda}^m), \tag{4d}$$

where

- Decisions: $\boldsymbol{\lambda}^m$ matching; $\boldsymbol{\lambda}^o_{\mathcal{P}}$ dispatching
- Equilibrium variable $\mathbf{m}$: statuses of followers in the system
- $\mathcal{S}_e$: equilibrium set, dependent on $\boldsymbol{\lambda}$
- $\mathcal{F}$: feasible sets, m for matching, d for dispatching

Network flow conservation and SUE

Transition of drivers (loop of vacant–pickup–delivery–vacant–. . .):



Stochastic user equilibrium, choice probability equals actual flow:

$$\mathbf{p}_{pq}^x \equiv \boldsymbol{\lambda}_{pq}^x \iff \text{SUE convex program/fixed-point system},$$

where $\mathbf{p}$ is agents' choice probability (e.g., logit), $\boldsymbol{\lambda}$ is the realized flow

Network equilibrium conditions cont'd

$$\mathbf{A}\boldsymbol{\lambda}^{o}_{\mathcal{P}} = \mathbf{q},$$ Supply–demand

$$\mathbf{q} = g(\mathbf{m}, \boldsymbol{\lambda}^{m}, h(\mathbf{q})),$$ Demand–delivery path flow–demand loop

$$\sum_{m} \lambda^{m}_{vm} = \sum_{c} \lambda^{v}_{cv}(\mathbf{m}, \boldsymbol{\lambda}^{m}, \boldsymbol{\lambda}^{o}_{\mathcal{P}}),$$ Flow conservation

$$\sum_{v \in \mathcal{N}^{match}_{m}} \lambda^{m}_{vm} = \sum_{i \in \mathcal{I}_m} \lambda^{o}_{\mathcal{P}_{m,i}},$$ Flow conservation

$$N_F = \sum_{m,\mathcal{P}_{m,i}} h_{m\mathcal{P}_{m,i}} \lambda^{o}_{\mathcal{P}_{m,i}} + \sum_{c,v} h_{cv} \lambda^{v}_{cv}(\mathbf{m}, \boldsymbol{\lambda}^{m}, \boldsymbol{\lambda}^{o}_{\mathcal{P}}) + \sum_{m,v} h_{vm} \lambda^{m}_{vm} + \sum_{v} m_v$$

Fleet conservation

Solution algorithm

A coordinate descent–based method

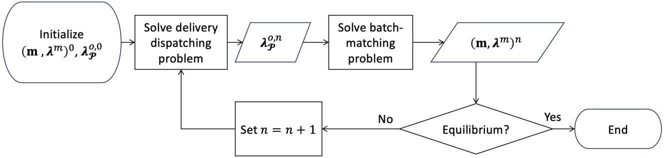


Figure 4: Flowchart of the solution algorithm.

[Decomposed program]

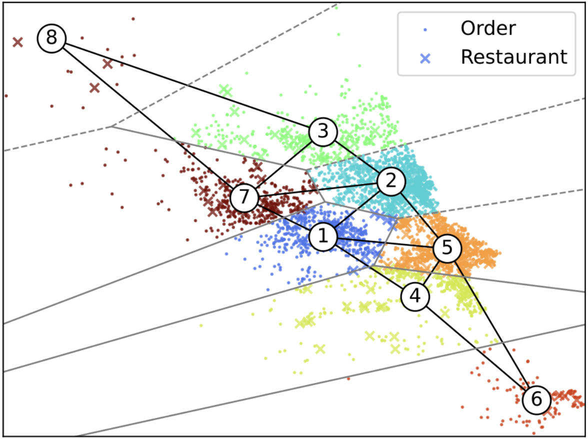
$$\boldsymbol{\lambda}_{\mathcal{P}}^{o,n+1} = \text{argmin}_{\boldsymbol{\lambda}_{\mathcal{P}}^{o}} \{ f_d(\boldsymbol{\lambda}_{\mathcal{P}}^{o}, \boldsymbol{\lambda}^{m,n}, \mathbf{m}^n) : \boldsymbol{\lambda}_{\mathcal{P}}^{o} \in \mathcal{F}_d(\boldsymbol{\lambda}^{m,n}, \mathbf{m}^n) \}, \tag{5a}$$

$$(\mathbf{m}, \boldsymbol{\lambda}^m)^{n+1} = \text{argmin}_{\boldsymbol{\lambda}^m} \left\{ \begin{array}{c} f_m(\boldsymbol{\lambda}^m, \mathbf{m}) : \mathbf{m} \in \mathcal{S}_e\left(\boldsymbol{\lambda}^m, \boldsymbol{\lambda}_{\mathcal{P}}^{o,n+1}\right) \\ \text{and } \boldsymbol{\lambda}^m \in \mathcal{F}_m(\mathbf{m}) \end{array} \right\}. \tag{5b}$$

The first program is linear; the second program's unknown variable grows polynomially with network size.

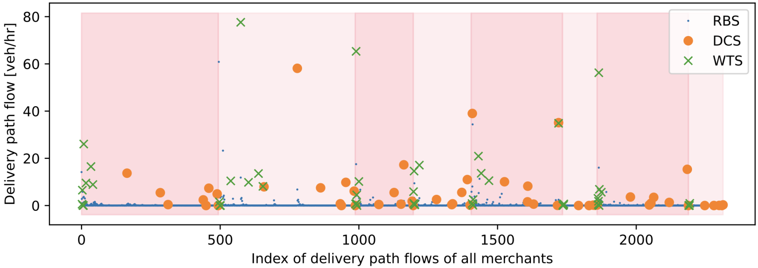
Numerical study on Grubhub data

Spatial distribution of demand; network clustered by K-means

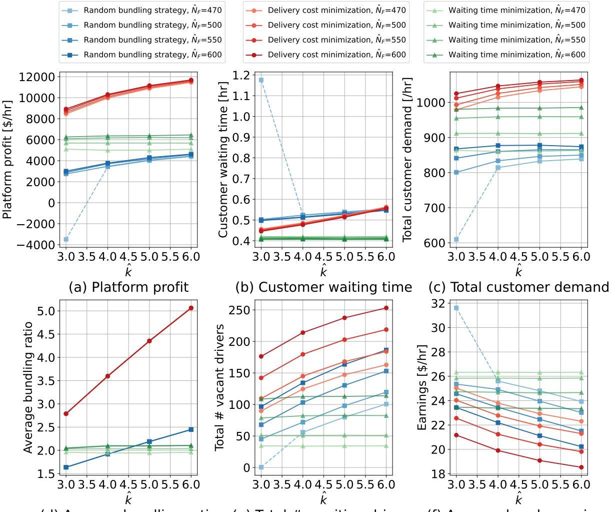


Delivery path flow distribution

- RBS: random bundling strategy (no intervention)
- DCS: delivery cost minimizing strategy
- WTS: waiting time minimizing strategy



Sensitivity analysis



(a) Platform profit (b) Customer waiting time (c) Total customer demand

(d) Average bundling ratio (e) Total # awaiting drivers (f) Average hourly earnings

Thanks for your attention

Future works

- Consider inter-depot traveling and driver behavior in a stylized model
- Modeling for network effects
- Unification of VRP-like and DPDP-like route approximations

Comments are welcome!